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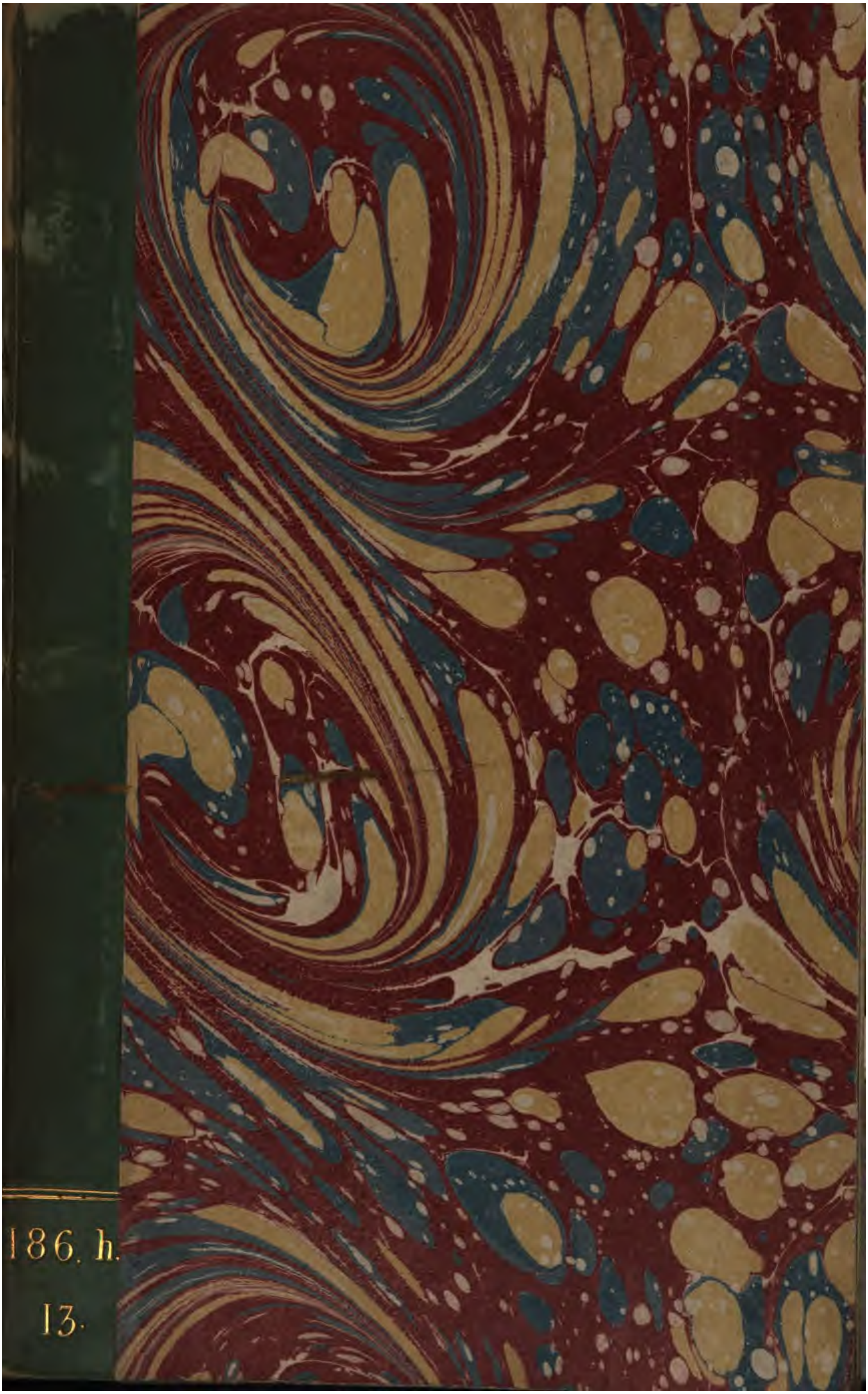

186. h.
13.

# DESCRIPTION

OF A

# CHRONOGRAPH.

WOOLWICH.
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# DESCRIPTION

OF A

# CHRONOGRAPH,

ADAPTED FOR MEASURING

THE VARYING VELOCITY OF A BODY IN MOTION THROUGH THE AIR,

AND

FOR OTHER PURPOSES.

BY

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BODLEIANA BIBLIOTHECA

[An Extract from the Proceedings of the Royal Artillery Institution, Woolwich.]

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# DESCRIPTION OF A CHRONOGRAPH.

THE resistance of fluids to the motion of solids is a problem of great practical and theoretical interest, because sometimes this resistance becomes a source of power, whilst at other times it is a large consumer of power which would otherwise be usefully applied. Thus the action of the wind upon sails is made to drive a mill or a ship. The wind acting upon the sails tends to drive the ship, and the resistance of the water is opposed to the progress of the ship. The resistance of the water to the motion of the oar, the paddle, or the screw, enables the rowers or the steam engine to drive the vessel. A very large part of the power developed in the locomotive is employed in overcoming the resistance of the air to the motion of the train. And quite recently it has been found that the friction of the tidal wave is probably slowly diminishing the velocity of the earth's rotation about its axis. We are dependent upon the resistance of fluids for our power to cross the ocean, for without that property we should not be able to use sail or oar, paddle or screw. Still little is known with accuracy respecting the laws of the resistance of fluids. It is extremely difficult to make satisfactory experiments on account of the great disturbance produced in the surrounding fluid, and as the mathematician knows neither the nature of this disturbance, nor the amount of resistance to be accounted for in particular cases, he is not able to find ground on which to base a satisfactory theory. For a history of what has been done, it will suffice to refer to the articles, "Ballistik" and "Widerstand," in Gehler's Physikalisches Wörterbuch, 1825 and 1842.

The resistance of the air to spherical balls, moving with high velocities, has been a subject of special interest for more than a century, because it was of practical importance to the science of gunnery, and because it offered a simple yet striking instance of the great resistance which a very rare medium would offer to a solid moving in it with a high velocity. Thus Robins[1] states that Dr Halley thought it reasonable to believe that the opposition of the air to large metal shot is scarcely discernible, although in small and light shot, he acknowledged that it ought and must be accounted for. Robins further states,[2] that a musket-ball, fired with a charge of half its own weight of powder, would leave the gun with a velocity of 1700 f.s., and that, with an elevation of 45°, its range would be 17 miles in a vacuum; whilst practical writers on the subject say that the range in air is short of half a mile. This resistance of the air $\rho$, depends upon the velocity $v$, upon the form, and upon the size of the moving body. When a body is at rest in air, the horizontal pressure tending to move it one way is just equal to the pressure tending to move it in the opposite direction. If the body be put in motion by any force, the pressure of the air tending to prevent motion is greater than it was before in that direction, and the pressure in the direction of the motion is less than it was before. The difference of the pressures of the air before and behind the body is called the resistance of the air to that particular form of solid moving with the given velocity: but as the pressure of the air in direction of motion decreases rapidly as the velocity increases, it is commonly neglected.

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[1] Gunnery. Preface, p. 48.

[2] p. 145.

Thus, $\rho$ is some function of the velocity $v$, as $f(v)$, such that when the body is at rest, or $v = 0$, we have $\rho = f(v) = 0$. It is usual to assume that $\rho$ can be expanded in a series of the form

$$\rho = av + bv^2 + cv^3 + dv^4 + ev^5 + \&c.$$

where $a$, $b$, $c$, &c. are to be determined by experiment, and are dependent on the form and size of the moving body.

Newton's experiments gave $\rho = bv^2$, and $\therefore\ a = c = d = \&c. = 0$,

Hutton's gave............... $\rho = av + bv^2$, and $\therefore\ c = d = \&c. = 0$,

Didion's gave ............... $\rho = bv^2 + cv^3$, and $\therefore\ a = d = \&c. = 0$,

Colonel Mayevski's gave... $\rho = bv^2 + dv^4$, and $\therefore\ a = c = \&c. = 0$,

It is possible that the resistance of the air, for a limited range of variation of velocity, may be tolerably represented by one or more of the above formulæ, but we must be careful not to assume that these formulæ must apply to all velocities of the same body, or that the same law will hold for all forms of bodies. Careful experiments must be made so as to discover some empirical law connecting the resistances of the air to bodies of simple forms and the velocities with which they move. When this is well done, all that is needed for practical use will have been obtained. But, beyond this, the mathematician will have been provided with facts which may serve as tests of the soundness of his theoretical investigations, and in the end he may succeed in forming a theory of resistances on mathematical principles, and thus replace our empirical formulæ with one that is true and general.

Before the discovery of electro-magnetism, the ballistic pendulum of Robins and Hutton afforded the only known practical means for determining the velocities of projectiles and the resistance to their motion exerted by the air. This instrument is so well known that there is no necessity for entering into a minute description of all its parts, and of the precautions to be taken in the use of it. The shot is fired into a body at rest much heavier than itself, to which it communicates motion, so that the two move on together with a common velocity which is capable of being measured. Towards the end of the last century numerous experiments were made with the ballistic pendulum, by Hutton,[1] at Woolwich, with 1, 3, and 6 lbs. spherical balls. Their velocities were measured at distances varying from 30 to 430 feet from the gun. Experiments were also made at Metz with the ballistic pendulum in 1839, &c. with balls of about 8, 12, and 24 lbs.[2] It was found that at 15m. (49 ft.) from the muzzle, the blast from the gun was very sensible. The velocities of the shot were determined at 15, 40, 65, and 90 metres (49, 148, 197, 295 ft.) from the gun. Beyond 295 feet the gun was not sufficiently accurate. Thus the law of the resistance of the air, deduced from the later and more careful experiments, depends upon the loss of velocity of spherical balls moving, at most, through 246 feet. But it is possible to make only one observation on each round. Therefore if it be desired to find the velocity lost by a ball in moving from 15 to 90 metres from the gun, one shot must be fired at the pendulum at a distance of 15 metres, and another at a distance of 90 metres; and, besides the errors in the measurement of the velocities at the moment of impact, there will be a doubt whether the velocity of the second ball, at 15 metres from the gun, was the same as that of the first shot at the same point. The initial velocity therefore enters into the question, and the only thing that can be done in such a case is, to make *numerous* experiments and trust to the *mean* of the results.

Hutton concluded from his experiments that the resistance of the air to a spherical ball $2R$ inches in diameter, was expressed by the formula,[3]

$$\rho = (\cdot 0000073\, v - \cdot 0015)\, v\, (2R)^2.$$

M. Piobert[4] re-calculated the experiments, and deduced the law,

$$\rho = 0\cdot 030586\ (1 + \cdot 0025\, v)\, v^2 \pi R^2 \text{ in French measures.}$$

Afterwards M. Didion[5] examined them and obtained the law,

$$\rho = 0\cdot 027\, (1 + \cdot 00257\, v)\, v^2 \pi R^2,$$

and M. Didion[6] gives as his final result, deduced from the Metz experiments,

$$\rho = 0\cdot 027\, (1 + \cdot 0023\, v)\, v^2 \pi R^2,$$

concluding his "Lois de la Résistance" with the remark on this formula: "Elle peut donc être appliquée avec confiance au tir de tous les projectiles *sphériques* en usage." It is quite plain that Hutton's experiments could not indicate any particular law with great precision since such varied ormulæ have been deduced from them.

The objections to this mode of experimenting are numerous and self-evident. As the weight of the ball is increased, the vibration and shock caused in the pendulum must give rise to large errors. With the increased distance, there is the difficulty of striking the pendulum so that there may be no impulse on the axis of suspension, and no tendency to twist about a vertical axis. Further, as experiments have been commonly made, the shot were not removed from the receiver until a certain number of rounds had been fired, and thus the positions of the centre of gravity and centre of percussion were in a state of constant variation. There was also the uncertainty arising from the necessity for assuming that the *average* initial velocities were the same for the same charge and ball for different distances of the receiver. The large guns now made cannot be experimented on with the ballistic pendulum in the ordinary way.

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[1] Hutton, Tracts, 1812.

[2] Didion, Traité de Balistique, 1848; 2nd Edition, 1861. Lois de la Résistance de l'Air, 1857.

[3] Hutton, Tract 37, p. 229.

[4] Didion, Traité, 1848, p. 44.

[5] Didion, Traité, 1861, p. 64.

[6] Didion, Traité, 1848, p. 52; 1861, p. 66; Lois, 1857, p. 79.

I have been particular in referring to dates in what precedes, because M. Didion's *Traité de Balistique* is our standard work on the subject. The theory of the edition of 1861 is practically the same as that of 1848, depending, not only upon the same law of the resistance of the air, but upon the same numerical formula of resistance. The tables to facilitate the practical application of the theory are also, to all intents, the same. We must therefore date this theory 1848, before the Crimean war, and before the introduction of rifled ordnance and projectiles of a cylindrical form. It is true that M. Didion does make some slight reference to small elongated projectiles, diameter 0m.119 (4·7 in.) and length 0m.240 (9·4 in.) for he remarks, "On reviendra, section ix., sur les résistances de divers genres que l'air fait éprouver aux balles oblongues; mais *en attendant que des expériences plus précises aient fourni des résultats plus certains,* nous admettrons pour les balles oblongues pleines et pour les boulets oblongs, un coefficient de résistance égal aux deux tiers de celui de la balle sphérique, c'est-à-dire $A=0{\cdot}081$ et $\rho'=0{\cdot}018\,(1+0{\cdot}0023v)$; il en sera les trois-quarts pour les balles creuses, comme celle du modèle 1859, c'est-à-dire égal à 0·020. On adoptera, pour les boulets oblongs de campagne et de siége, le même coefficient $A=0{\cdot}018$."[1] It is plain, however, that the resistance of the air to elongated shot depends greatly upon the form of the shot, and that something far more precise than the above is required.

It is remarkable that Hutton concluded from his experiments that, for every 100 feet added to the velocity of a shot, there was an increasing addition made to the resistance of the air up to a velocity between 1600 f.s. and 1700 f.s., where it attained a maximum. Thus, in his table of the air's resistance to a ball of 2 inches in diameter, we find[2]

| vel. f.s. | Resistance in oz. | $\Delta_1$ | $\Delta_2$ |
|---|---|---|---|
| 1300 | 683·3 | | |
| 1400 | 811·5 | +128·2 | +7·4 |
| 1500 | 947·1 | +135·6 | +4·2 |
| 1600 | 1086·9 | +139·8 | +1·7 |
| 1700 | 1228·4 | +141·5 | −1·3 |
| 1800 | 1368·6 | +140·2 | −3·1 |
| 1900 | 1505·7 | +137·1 | −5·0 |
| 2000 | 1637·8 | +132·1 | |

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[1] Didion, Traité, p. 74, 1861.

[2] Hutton, Tract 37, p. 218.

This asserted maximum is in itself very improbable, and M. Didion[1] declares that there is none deducible from Hutton's experiments. Hutton's attempt to account for this supposed maximum, by reference to the velocity with which air rushes into a vacuum, is extremely fanciful—because the after-pressure is not great to begin with, it diminishes rapidly as the velocity is increased, but perhaps never entirely vanishes.

Professor Wheatstone applied electricity to measure the velocities of shot in 1840. Afterwards, several changes were made in his instrument. Drawings were shewn in Paris, in May, 1841, and a copy was prepared for Colonel Konstantinoff; but he desired to have an instrument by which he could determine the *velocity of a shot at different points of its path.*[2] M. Breguet afterwards constructed an instrument for this purpose, the performance of which will be further noticed hereafter. This led to a dispute between Professor Wheatstone and M. Breguet, respecting their claims to the invention of the chronograph.[3] Pouillet attempted to determine short intervals of time on the supposition that the deflection of a magnet needle is proportional to the strength of a galvanic current, and to the time it is acted upon by the current. Hipp objected to Professor Wheatstone's arrangement, and made use of a clock going uniformly to measure his time. It does not appear that any of these chronographs have come into use, or that they have given results of any importance.

The electro-ballistic pendulum of Major Navez was invented about 1848, but was brought prominently into notice only in 1855. Very various opinions have been expressed respecting the value of this instrument. The Americans tried it and rejected it on account of the irregularity of its indications. The French used it in 1856, 1857, and 1858, at Metz, to complete their researches on the resistance of the air. "Les résultats n'ont pas été formulés, ils font voir qu'aux vitesses moyennes la valeur de $\rho'$ est égale à celle qui a déjà été donnée, mais que pour des vitesses plus petites la résistance diminuerait *beaucoup plus rapidement* que par la formule déjà obtenue (Art. 52, éq. 7), et qu'elle décroîtrait même trop rapidement pour être dès maintenant admise".[4] Experiments with small arms were continued in 1857 and 1858, with new precautions, on which M. Didion observes: "Elles ont présenté, comme les précédentes, une diminution trop rapide dans les valeurs de $\rho'$, et les résultats *n'ont pas donné une formule qu'on puisse appliquer au tir avec une entière confiance*"[4]. Others who have used the instrument have expressed a very high opinion of its accuracy. It is undoubtedly a great improvement upon all instruments that preceded it for rough practical work, but it is wanting in that precision we are entitled to ask for when electricity is brought to our assistance. It may well be doubted whether anything has been contributed to our knowledge of the laws of the resistance of the air by Major Navez's electro-ballistic pendulum, or by the modifications of it by De Brettes, Vignotti, Leurs, &c. In 1861, it is plain that M. Didion gave the preference to the results of the old ballistic pendulum.

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[1] Didion, Traité, p. 64, 1861.

[2] Müller, Berichte, p. 856, 1848.

[3] Moigno, Traité de télégraphie électrique,

[4] Didion, Traité, 1861, p. 71,

The electro-ballistic pendulum consists of a short pendulum capable of revolving about a horizontal axis through an angle of 75° on each side of the vertical through the point of suspension. As a preparation for the experiment, the bob is raised to its highest position on one side, where it is retained by the attraction of an electro-magnet, whose magnetism is excited by a galvanic current so arranged that, when the shot arrives at the first screen, it breaks the wire through which the galvanic current was passing, and consequently sets the bob free. When the shot reaches a second screen it breaks a second galvanic current, and so destroys the power of a second electro-magnet which had hitherto supported a small weight. This weight now drops, and in its fall *completes* a third galvanic stream which excites a third electro-magnet, and so clamps a light index which had been travelling with the pendulum from its position of rest. The index reading is afterwards taken, and is supposed to give the angle through which the pendulum had moved when the third galvanic stream was closed. An instrument is used from time to time to break simultaneously the first and second galvanic streams, in order to find the time lost by the falling of the weight, by the electro-magnets, &c. One instrument is capable of measuring only *one short* interval of time. The time of describing the various arcs of the instrument are calculated from theory, which neglects friction, temperature, and the resistance of the air, and depends upon the observed time of a mathematically small oscillation of the pendulum. I believe Major Navez directs that not less than 1500 vibrations should be counted. Supposing there are three to the second, this experiment would last 500 seconds, or 8⅓ minutes, an extremely short time in which to determine the rate of an exact measurer of time. If the 1500 *sufficiently small* vibrations were correctly counted *consecutively*, and if the time was noted by a good chronometer, there would be an error, greater or less, at the beginning and end of the counting. If the vibrations were counted at two or more times, then there would be twice as many small errors. The screens are formed of fine copper wire through which the galvanic currents are made to circulate. The wire is fine in order that it may not obstruct the motion of the ball, and of copper, for the sake of its good conducting power. But copper wire is very liable to stretch before it breaks. Even the wind or the blast accompanying the ball will interfere with the uniformity of its tension. Hence the objections that strike one on looking at such an instrument, are:—

(1) The arrangement of the galvanic streams, varied and complicated.

(2) The determination of the time table not satisfactory.

(3) The wires of the screens are almost certain to be unequally stretched.

(4) The instrument affords no means of checking the accuracy of its indications.

(5) The necessity to employ two or more independent instruments to find the resistance of the air.

Some of these objections might be removed, but the last two appear unavoidable.

Recently a chronographe électro-balistique has been invented by M. P. le Boulengé, Lieutenant d'Artillerie, which has been tried and compared with Major Navez's electro-balistic pendulum, by M. Melsens, Membre de l'Académie Royale de Belgique. He states that M. Boulengé, in his memoir, renders what is just to Major Navez, remarking, "Cet officier en effet a donné, le premier, à l'Artillerie un appareil remplissant toutes les conditions exigées pour être utilement employé aux recherches balistiques; on sait que les essais faits en Angleterre, en France, en Allemagne par des savants illustres, par des militaires distingués, aidés d'ingénieurs et de fabricants habiles, avaient laissé sans solution pratique la question de la détermination des vitesses des projectiles pour tous les cas possibles, tant pour les armes portatives que pour les armes de fort calibre, tirant sous des angles variables."[1] Experiments were made with various powders to compare the two Belgian instruments. The velocity of the same projectile was measured by both instruments for a point 35 metres from the gun. The following list gives the mean velocities in m.s. of five or six rounds for each kind of powder as measured by the two instruments:[2]—

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Navez ...... | 373·9 | 376·1 | 369·6 | 371·0 | 367·1 | 375·1 | 368·7 |
| Boulengé... | 367·9 | 366·6 | 368·2 | 366·8 | 361·4 | 369·7 | 364·9 |
| Difference... | −6·0 | −9·5 | −1·4 | −4·2 | −5·7 | −5·4 | −3·8 |

| | | | | | | |
|---|---|---|---|---|---|---|
| Navez ...... | 359·1 | 352·7 | 369·7 | 368·2 | 366·7 | 351·4 |
| Boulengé... | 355·9 | 346·0 | 365·2 | 364·8 | 366·7 | 353·4 |
| Difference... | −3·2 | −6·7 | −4·5 | −3·4 | 0·0 | +2·0 |

Or, converting the differences of the means of the velocities, as determined by the two instruments, into feet-seconds, we find them: −19·7, −31·2, −4·6, −13·8, −18·7, −17·7, −12·5, −10·5, −22·0, −14·8, −11·2, 0·0 and +6·6. M. Melsens appears to give the preference to M. Boulengé's instrument, but he makes the remark, "Des expériences *novelles* et *nombreuses* diront où est la vérité absolue."[3]. Now it is really remarkable that two instruments with any pretensions to accuracy should give such contradictory results. Five or six shot are fired, the velocity in each case is measured by two instruments. The mean of the velocities given by one instrument is compared with the mean of the velocities given by the other, and they differ in one case 31·2 f.s., in another 22·0 f.s., in another 19·7 f.s., and so on. The difference of the *means* varies from −31·2 f.s. to + 6·6 f.s. Thus so long as instruments are used without check, there will always be uncertainties to be settled by new and more numerous experiments. It is the great fault of these instruments that, whilst they are something near the truth, they are not sufficiently exact to give the law of the resistance of the air. But this is the problem that must be solved by any instrument that pretends to accuracy. This must be regarded as the test of the value of any chronograph. Robins was not content merely to determine a list of initial velocities, for he at once proceeded to deduce the resistance of the air to projectiles moving with any velocity. Hutton did the same. Turn to Didion's Treatise, and there it is found that Chapter I. is devoted to the consideration of projectiles moving in a vacuum, but in Chapter II., when he comes to the consideration of a practical case, he is met by the old difficulty of the *amount* and *law* of the resistance of the air. He also requires to know in what way, and to what extent, the resistance of the air is affected by the *form* and *size* of the moving body. Now it is remarkable that the electro-ballistic pendulum should have been in such general use for more than ten years, and that no satisfactory attempt should have been made to confirm, or otherwise, the laws determined by the old ballistic pendulum for round shot, and to extend them to cylindrical bolts with variously-formed heads. So far as the comparison of the two pendulums is known to have been attempted, we have seen, on the authority of M. Didion, that they give a different law, and that he considers the results of the old ballistic pendulum the more satisfactory of the two. And I am the more inclined to believe that nothing had been attempted with success up to last year from the remarks made by Major Navez in his "*Considérations sur les Expériences de Balistique en ce qui concerne la Mesure du Temps,*" 1865. In that tract, he explains in general terms the method to be pursued in order to determine the resistance of the air. He recommends that the screens should be placed as much as 100 metres apart. He finds that M. Boulengé's chronograph cannot measure an interval of time exceeding 0″·1; that of Colonel Leurs cannot exceed 0″·227964. He thinks it would be advantageous to have a superior limit of 0″·32, but contents himself with 0″·2. Major Navez remarks in his introduction: "Nous avons souvent entendu, nous entendons encore chaque jour, des officiers d'artillerie regretter que l'on ne connaisse pas, d'une manière plus précise, la véritable expression de la résistance de l'air quant à sa forme analytique et à sa valeur."[1] And again, "Jusqu'à présent on connaît peu de chose sur la loi de la résistance que l'air oppose aux projectiles allongés; plus tard, lorsque des expériences complètes auront fait découvrir la véritable loi de cette résistance, on trouvera probablement le procédé d'interpolation par arc de cercle bien imparfait. C'est pourquoi nous tenons à établir qu'il s'agit seulement ici d'approximations, grossières peut-être, mais suffisamment approchées de la vérité pour éclairer nos investigations."[2] And the following remark is worthy of particular attention: "Quand on examine en masse les résultats obtenus dans les expériences sur les vitesses conservées par nos projectiles allongés, à différentes distances de tir, on doit se trouver fort satisfait, en ce sens, que de ces résultats ressort, à l'évidence, la bonne conservation de la force vive. *Mais si on se livre à un examen plus minutieux, on voit que ces résultats ne sont pas, en général, assez réguliers pour qu'on puisse les employer à établir la loi de la résistance que l'air oppose aux nouveaux projectiles.*"[3] The perusal of the "Considérations" may be strongly recommended to any one who wishes to understand the true state of the case and the difficulties that have to be overcome. I have always held, and still hold, that as nothing really good has been effected by the ordinary chronographs, so nothing can be done by them. Rather than attempt to improve upon old methods, I decided to apply a known method of experimenting which was new to this branch of science.

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[1] Melsens, Rapport sur un Chron., 1864. p. 5.

[2] Ibid. Rapport, p. 21.

[3] Melsens, p. 24.

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[1] Considérations, p. 4. [2] Ibid. p. 17. [3] Ibid. p. 34.

After a due consideration of all circumstances of the case, it appeared that the following conditions must be satisfied by a chronograph worthy of perfect confidence :—

(1) The time to be measured by a clock going uniformly.

(2) The instrument to be capable of measuring the times occupied by a cannon ball in passing over at least *nine* successive *equal* spaces.

(3) The instrument to be capable of measuring the longest known time of flight of a shot or shell.

(4) Every beat of the clock to be recorded by the *breaking* of the same galvanic current, and under precisely *the same conditions*.

(5) The time of passing each screen to be recorded by the momentary *interruption* of a second galvanic current, and under precisely *the same conditions*.

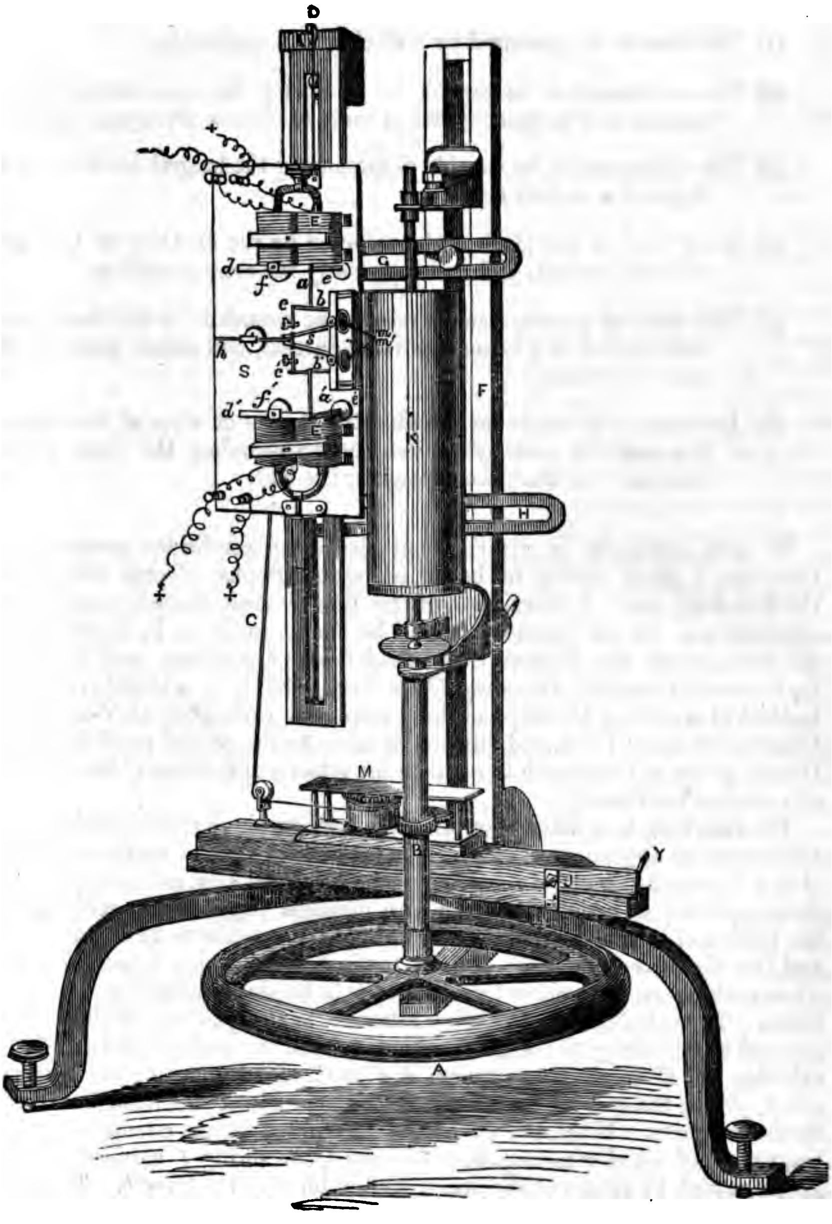
(6) Provision to be made for keeping the strings or wires of the screens in *a uniform state of tension*, notwithstanding the force of the wind and the blast accompanying the ball.

To gain assistance in carrying out the above conditions practically, I consulted a great variety of books on chronographs, electric telegraphs, clock making, &c. A description of one of the most recently constructed chronographs for an Observatory will be found in C. A. F. Peter's *Ueber die Bestimmung des Langen unterschiedes zwischen Altona und Schwerin.* In Loomis's *Practical Astronomy* (New York, 1855), it is stated that this method of recording transits had been employed exclusively at Washington Observatory since 1849, and allusion is made to the conical pendulum then (1855) in use at Greenwich to regulate the velocity of rotation of the cylinder of a similar instrument.

The following is a description of the chronograph as constructed, and of various useful appendages. Fig. 1 gives a general view of the chronograph. *A* is a fly-wheel capable of revolving about a vertical axis, and carrying with it the cylinder *K*, which is covered with prepared paper for the reception of the clock and screen records. The length of the cylinder is 12 or 14 inches, and the diameter 4 inches. *B* is a toothed-wheel which gears with the wheelwork *M* so as to allow the string *CD* to be slowly unwrapped from its drum. The other end of *CD* being attached to the platform *S* allows it to descend slowly along the slide *L*, about ¼ inch for each revolution of the cylinder. *E*, *E'* are electro-magnets; *d*, *d'* are frames supporting the keepers; and *f*, *f'* are the ends of the springs which act against the attraction of the electro-magnets. When the current is interrupted in one circuit, as *E*, the magnetism of the electro-magnet is destroyed, the spring *f* carries back the keeper, which by means of the arm *a* gives a blow to the lever *b*. Thus the

**marker *m* is made to depart from the uniform spiral it was describing. When the current is restored the keeper is attracted, and thus the marker *m* is brought back, which continues to trace its spiral as if nothing had happened. *E'* is connected with the clock, and its marker *m'* records the seconds.**

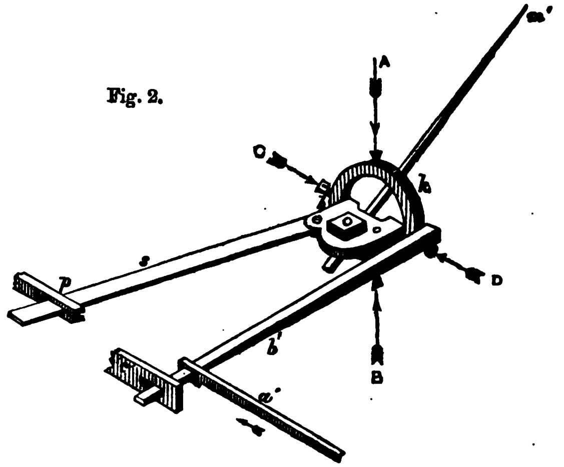
Fig. 1.



$E$ is connected with the screens, and records the passage of the shot through the screens. By comparing the marks made by $m$, $m'$ the exact velocity of the shot can be calculated at all points of its course. The slide $L$ is fixed parallel to $F$ and the cylinder $K$ by the brackets $G$, $H$. $Y$ is a screw for drawing back the wheelwork $M$, and $J$ a stop to regulate the distance between $M$ and $B$. The depression of the lever $h$ raises the two springs $s$, which act as levers, and bring the diamond points $m$, $m'$ down upon the paper. When an experiment is to be made, care is taken to see that the two currents are complete. The fly-wheel $A$ is set in motion by hand, so as to make about three revolutions in two seconds. The markers $m$, $m'$ are brought down upon the paper, and after four or five beats of the clock the signal to fire is given, so that in about ten seconds the experiment is completed, and the instrument is ready for another.

Fig. 2 gives a full-sized view of one of the markers, shewing the way in which it is moved. The depression of the lever $h$ (Fig. 1), raises $p$, and thus the lever $s$, which is formed of watch-spring wire, brings down $m'$ to the paper, and keeps it gently in contact. This motion takes place within the circle $k$, about an axis $CD$. $a'$ is an arm connected with the electro-magnet. When the magnetism in $E'$ is destroyed, $a'$ begins to move away, and when it has moved a short distance it strikes the lever $b'$ a sudden blow which carries it as far as the hole in the stop $c'$ will allow it to move. The The lever $b'$ is rigidly connected with the circle $k$, which is capable of moving about an axis $AB$. This motion is communicated to $m'$, which describes a very short arc of a circle about a point in $AB$. The arrangement is so made that when either of the markers $m$, $m'$ is making a record, it has a motion which may be resolved partly in direction of the motion of the paper under it, and partly in a direction perpendicular to this. Thus records are obtained which can be read off by scale with great nicety.

Fig. 2.



The pendulum of a half-seconds clock strikes once each double-beat a very light spring, and so interrupts the galvanic current in $E'$ once a second.

Figs. 3 and 4 give the details of the screens. Fig. 3 represents a piece of board 1 inch thick and 6 or 7 inches wide, and rather longer than the width of the screen to be formed. Transverse grooves are cut at equal distances, something less than the diameter of the shot, as shewn in the diagram.

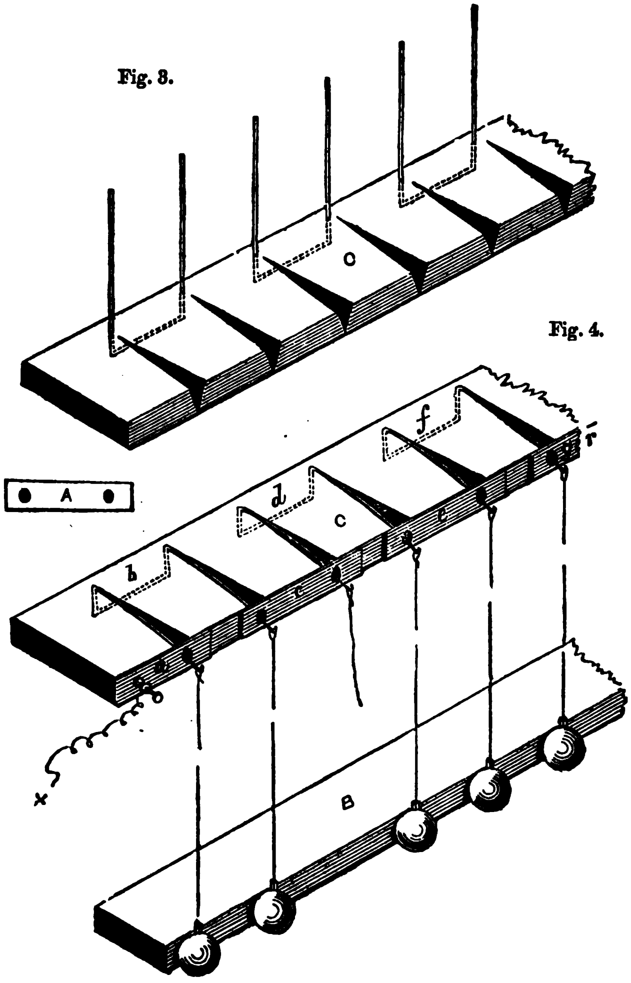


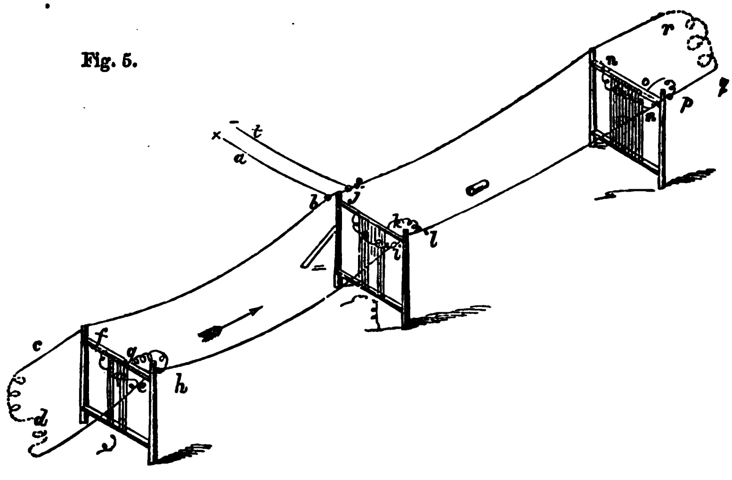
Fig. 3.

Fig. 4.

Staples of hard brass spring-wire (No. 14 or 15), are fixed with their prongs in the continuation of the grooves. Pieces of sheet copper *A* are provided, having two elliptical holes, the distance of whose centres equals the distance of the grooves. The pieces of copper *A* are used to connect each wire steeple, as *C*, with its neighbour on each side. Thus, Fig. 4, *a*, *c*, *e*, *g*, &c., represent these copper connections put in their places and holding down the wire spring, which, when free, are in contact with the tops of the holes; but, when properly weighted, they rest on the lower edge of the holes. Thus the copper *c* forms a connexion between the staples *b* and *d*; the copper *e* joins *d* and *f*, and so on. A galvanic stream will therefore take the following course, whether the springs be weighted or unweighted: copper *a*, brass *b*, copper *c*, brass *d*, copper *e*, brass *f*, copper *g*, &c. The current will only be interrupted when one or more threads have been cut and the corresponding spring is flying from the bottom to the top of its hole. About $\frac{1}{50}$th of a second is required for the complete registration of such an interruption, the spring traversing about half an inch. The shelf *B* is placed for the weights to rest against, partly to prevent them from being carried forward by the shot, but chiefly to prevent the untwisting of the threads which support the weights. The weights used were about 2 lbs. each, and the strength of the sewing cotton for supporting them was equal to a stress of about 3 lbs., which was sufficient to withstand a tolerably strong wind. As the weights were equal the threads were kept equally stretched.

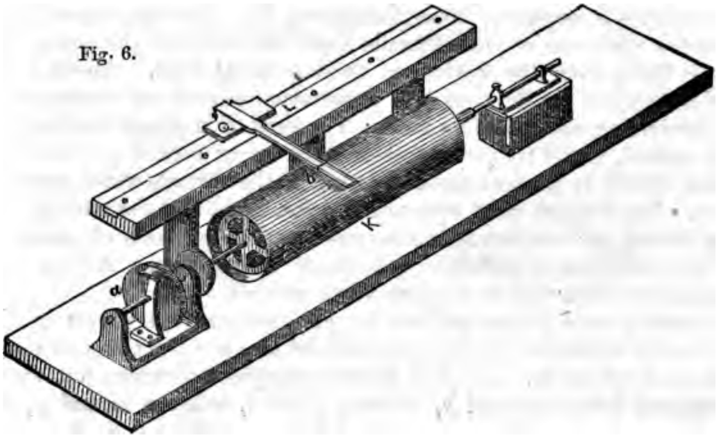
The arrangement of the screens for an experiment is shown in Fig. 5. The wires for conveying the galvanic current are, like the common telegraph wire, carried on posts. *abc* is a continuous piece of wire; but there are interruptions between *e* and *h*, between *i* and *l*, between *m* and *p*, &c., in order to make the galvanic current circulate through the screens. The course of the galvanic current is + *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i*, *j*, *k*, *l*, *m*, *n*, *o*, *p*, *q*, *r*, *s*, *t*. The ends *a*, *t*, are connected with the instrument and battery. The shot, being fired through the screens, in passing cuts one or more threads at each screen, so that corresponding to the instant at which the shot passes each screen there is an interruption of the galvanic current, and a simultaneous record on the paper.

Fig. 5.



When the cylinder is filled with spirals, that is after five or six shots, it is transferred to the instrument, Fig. 6, where *a* is a circle divided into 300 equal parts, and the division is carried to 3000 by the help of a vernier. A small T-square, having a fine edge at *b*, moves along a brass straight-edge *L*, adjusted so as to be parallel to the axis of the cylinder. The mark *b* is carefully placed opposite each record on the paper by means of a tangent screw (not shewn in the figure), and the vernier is read.

Fig. 6.



The clock goes on breaking the galvanic circuit every swing of the pendulum, whether the marker *m* be in contact with the paper or not—consequently, whatever be the loss of time in the action of the marker, we may fairly suppose it to be constant. But it has been objected that the current having been circulating through the screens for several seconds, or even minutes, without interruption before the shot is fired, the records at the first and the following screens are not made under the *same* conditions. A careful inspection of the results actually obtained shewed that this objection had no foundation to rest upon for that particular experiment; still it seemed desirable to get rid of the objectionable arrangement lest an irregularity might make its appearance in some important experiment. The first screen was placed 120 feet from the gun, the second at 240 feet, and so on. One preparatory break would be obtained by placing a screen at the mouth of the gun; but in addition to this it appears well to introduce the ordinary self-acting contact breaker, made to beat as near as may be the intervals of time to be measured. The raising of a spring lever interrupts the main current of galvanism through the screens. The insertion of a pin to keep up the lever, re-opens a passage for the screen galvanic current through the contact breaker; this may be made also to ring a bell in the instrument room, to give notice that all things are ready for the experiment. The fly-wheel is then put in motion, the signal to fire is given; the pulling of the lanyard withdraws at once the pin, restores the main current, and fires the gun.

The construction of the chronograph was commenced in August, 1864; it was ready for trial in June, 1865. It received its first *partial* trial before the Committee on Gun Cotton in July, 1865, in conjunction with Major Navez's Electro Ballistic Pendulum. The instruments gave a nearly constant difference of 20 f.s. in velocities of about 1500 f.s., and indicated a much greater consistency in the results than in those obtained by M. Melsens when engaged in comparing the two instruments of Major Navez and M.

Boulengé. The chronograph remained at the proof butts from July to November, 1865, when it was taken down to Plumstead Marshes and placed in a splinter-proof, where it remained about a fortnight. Its powers to withstand damp and dust were well tested in this manner. Eighteen shots were fired in all. Two of them were fired by mistake when the cylinder was stationary. One shot carried away a screen, another cut the conducting wire at the second screen, &c. I am able, however, to give a good account of eleven out of the eighteen shots fired on November 23, November 29, and December 12, 1865.

Before proceeding to give an account of the experimental trial of the chronograph, it is necessary to explain at some length the mode of dealing with the clock and screen records, because the absence of any provision to secure uniformity of rotation of the cylinder has appeared to many a great defect. The fact that the variation of velocity of a pendulum, falling from rest through a considerable angle, used in so many instruments, is enormously greater, seems to have been entirely overlooked. The reason for not attempting to secure uniform rotation in the present case was this: the thing has been often attempted, and, so far as I know, never satisfactorily accomplished, when such small intervals of time as the one-hundredth or one-thousandth of a second were of importance. It is sufficient to refer to the chronograph constructed by M. Breguet for Colonel Konstantinoff. M. Gloesener[1] appears to be satisfied with the approach to uniformity of rotation made by that instrument, and gives four experiments in support of his favourable opinion. It will be sufficient to copy the second experiment, which gives the times occupied in making five revolutions in succession:—

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 2″·05 | 1″·95 | 1″·90 | 1″·80 | 1″·60 | 1″·90 | 1″·90 | 1″·80 |
| 2″·05 | 2″·00 | 1″·90 | 1″·80 | 1″·90 | 1″·80 | 1″·50 | 2″·30 |
| 2″·00 | 1″·80 | 1″·80 | 1″·90 | 1″·80 | 1″·60 | 1″·80 | |
| 2″·05 | 2″·00 | 1″·80 | 1″·90 | 1″·90 | 1″·80 | 1″·80 | |

giving a mean time of revolution of 0″·362.

As this result was very far from satisfactory to me, and I had no hope of succeeding so well as M. Breguet, it was quite plain that the difficulty of making the cylinder revolve uniformly must be avoided in the construction of a good chronograph for experiments on gunnery. As already explained, in most, if not in all instruments used in Observatories for recording observations by the aid of electricity, a marker connected with the clock records the seconds. If this were done, it is plain that the length of spiral described in successive seconds would give the rate at which the cylinder was moving. The introduction of a heavy fly-wheel would prevent sudden variations of velocity. The axis was placed in a vertical position, in order to reduce the friction as much as possible, and to avoid errors that might be caused by one side of the fly-wheel being heavier than the other. All unnecessary resistances, all disturbing causes must be removed. It may then be assumed that the cylinder will lose its angular velocity according to some law, which law can be determined from the intervals between five or six successive records of the beats of the clock. The weight of the marking apparatus and electro magnets does, in some measure, act as a driver of the fly-wheel and cylinder, and so tends to keep up the velocity; but it was not introduced for that purpose, and its effect is slight. The *gradual* loss of angular velocity of the cylinder gives a little trouble in the calculation when *extreme* accuracy is desired, but creates not the slightest difficulty to the mathematician. The subject now must be divided into two parts; (1) where a thoroughly trustworthy velocity of a projectile is required, and (2) where the utmost attainable precision is desired, with a view to determine the exact amount and law of the resistance of the air to projectiles of various forms and sizes.

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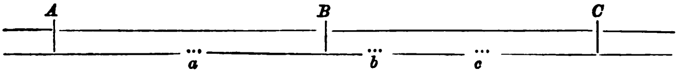
[1] Traité général des applications de l'électricité, &c., I, 374.

In the first case, the intervals between at least *three* successive beats of the clock should be measured, so as to extend beyond the screen marks on both sides. *Three* screens are the least that can be used with safety, which may be placed 60 or 70 feet apart, or at greater equal distances, if possible. By taking *three* records from the clock, and *three* from the shot, we have a complete check. If the records be found to be consistent, and if $A$, $B$, $C$, represent the beats of the clock, and $a$ the position of the three screen marks, then the length $AB$ must be taken to represent a second, and the distance from the first to the third mark at $a$ would represent, very nearly on the same scale, the time occupied by the shot in passing from the first to the third screen—the use of the middle screen being merely to test the reliability of the screen records. The only calculation is comprised in the proportion—velocity in f:s : distance between first and third screen :: $AB$ : distance between first and third record at $a$. If the three screen records fall at $b$ near $B$, then $\frac{1}{2}$ ($AC$) must be taken as the representative of a second; or if they fall at $c$, then length of $BC$ must be used to represent a second. The velocity so determined would be thoroughly trustworthy, and sufficiently accurate for all practical purposes.



In the second case, where the utmost attainable accuracy is desired, it becomes necessary to have more numerous equidistant screens spread over a greater distance. The screens are placed at *equal* distances because that facilitates calculation, and enables us to judge whether the records are *consistent*. This principle should be carefully observed in all experiments. Thus, if it be required to find how far the initial velocity depends upon the charge of powder, vary the charge by *equal* quantities. If several rounds are to be fired with each particular charge, begin with the highest or lowest, and fire one round of each charge up to the lowest or highest, and so on over again till the experiment be completed. If it be desired to find range tables corresponding to various elevations of the gun, fire one shot at 0°, one at 1°, one at 2°, and so on up to the greatest elevation, and repeat till the experiment is completed. If now the mean range for each degree of elevation be taken, these ranges will follow some kind of law, and after small adjustments have been made by differencing, the ranges for all intermediate elevations may be found by interpolation. For much valuable information on the mathematical treatment of such subjects, I must refer the reader to three papers by Mr Woolhouse, "On Interpolation, Summation, and the Adjustment of Numerical Tables."[1] He remarks, "If in the direct calculation of a table, the values to be tabulated depend upon a complicated function, difficult of calculation, considerable labour and corresponding risk of error may usually be avoided by computing the values for a *certain succession of equidistant intervals*, and afterwards determining by interpolation all other values that may be requisite to complete the table and adapt it to the practical purposes for which it is constructed. In such case the method is made use of as a powerful *auxiliary* at the option of the computer. If, however, the only available data are *given values* at certain *regular* intervals, and the *mathematical form of the function is unknown*, then the process of interpolation under some of its modifications, becomes *absolutely indispensable*, and its true application is necessarily of great importance where accuracy is required," p. 61. And again, "When a series of quantities which depend upon a *fixed law*, whether *known* or *implied*, follow each other in due order of succession, the general accuracy of their numerical values may be satisfactorily tested by observing the regularity of the progression of a suitable order of differences. If the tabular quantities are the results of calculation from a given formula with *equidistant* arguments, by differencing them up to a certain order, the existence of an isolated error, if one should exist, is thus prominently exhibited, and therefore speedily detected," p. 136. "If, for example, a number of errors, each of which may be of any magnitude, were promiscuously introduced into a page of a table of logarithms, or in any table computed mathematically according to a given formula or generic law which is *still inherent to the general mass of values*, the preceding rules, when applied to the differences would necessarily have the effect of practically eliminating the several errors and restoring the table to its original state of accuracy," p. 142.

The application of the above methods in this particular case is of a very simple kind, because the loss of velocity, both of the fly-wheel and of the shot, goes on very regularly during the short time we are concerned about them. The whole process is shewn applied to Round 1, and the results are given for all the other rounds. The manner of treating the varying velocity of rotation of the cylinder may be illustrated by reference to an ordinary table, say of logarithms. In this case there is not a uniform increase of logarithms corresponding to a uniform increase of numbers, and yet the tables are so arranged and provided with tables of differences, that there is not the slightest difficulty in finding the logarithm of any number, or the number corresponding to any logarithm within the limits of the table.

Suppose that the following logarithms of numbers have been obtained by independent calculation, some of which may be erroneous. They may be tested by taking successive differences, as in (I); then corrected so as to make the column $\Delta_2$ regular, and re-differenced as in (II).

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[1] The Assurance Magazine, Vol. XI. p. 61-68; 301-332; and Vol. XII. p. 136-176.

(I)

| Nos. | log | $\Delta_1$ | $\Delta_2$ | Corrections. |
|---|---|---|---|---|
| 1280 | 107 210 | + 3380 | | |
| 1290 | 110 590 | + 3353 | − 27 | |
| 1300 | 113 943 | + 3318 | − 35 | + 10 |
| 1310 | 117 261 | + 3313 | − 5 | − 20 |
| 1320 | 120 574 | + 3278 | − 35 | + 10 |
| 1330 | 123 852 | + 3254 | − 24 | − 1 |
| 1340 | 127 106 | + 3228 | − 26 | + 2 |
| 1350 | 130 334 | + 3205 | − 23 | − 1 |
| 1360 | 133 539 | + 3182 | − 23 | |
| 1370 | 136 721 | | | |

(II)

| log | $\Delta_1$ | $\Delta_2$ |
|---|---|---|
| 107 210 | + 3380 | |
| 110 590 | + 3353 | − 27 |
| 113 943 | + 3328 | − 25 |
| 117 271 | + 3303 | − 25 |
| 120 574 | + 3278 | − 25 |
| 123 852 | + 3253 | − 25 |
| 127 105 | + 3229 | − 24 |
| 130 334 | + 3205 | − 24 |
| 133 539 | + 3182 | − 23 |
| 136 721 | | |

It is at once evident from (I) that a correction of + 10 applied to 117 261, and a correction of −1 applied to 127 106, will introduce regularity into the column of second differences $\Delta_2$. These corrections are introduced in (II), where the differencing has been repeated. We cannot apply the common rule of proportional parts to calculate the logarithms of intermediate numbers, because the column of first differences $\Delta_1$ is so far from being constant. We can, however, find by interpolation the logarithms of 1335, 1345, 1355, &c., thus:—

(III)

| Nos. | log | $\Delta_1$ | $\Delta_2$ |
|---|---|---|---|
| ...... | | | |
| 1330 | 123 852 | + 1629 | |
| 5 | 125 481 | + 1624 | − 5 |
| 1340 | 127 105 | + 1617 | − 7 |
| 5 | 128 722 | + 1612 | − 5 |
| 1350 | 130 334 | + 1605 | − 7 |
| 5 | 131 939 | + 1600 | − 5 |
| 1360 | 133 539 | + 1594 | − 6 |
| 5 | 135 133 | + 1588 | − 6 |
| 1370 | 136 721 | | |

Still the column of first differences $\Delta_1$ is far from constant, and therefore the rule of proportional parts cannot be applied as (III) now stands. Let the interpolation be carried further, as below:—

(IV)

| Nos. | log | $\Delta_1$ | $\Delta_2$ |
|---|---|---|---|
| ...... | | | |
| 1345 | 128 722 | + 323 | |
| 6 | 129 045 | + 323 | 0 |
| 7 | 129 368 | + 322 | − 1 |
| 8 | 129 690 | + 322 | 0 |
| 9 | 130 012 | + 322 | 0 |
| 1350 | 130 334 | + 322 | 0 |
| 1 | 130 656 | + 321 | − 1 |
| 2 | 130 977 | + 321 | 0 |
| 3 | 131 298 | + 321 | 0 |
| 4 | 131 619 | + 320 | − 1 |
| 1355 | 131 939 | | |

On referring to a table of seven-figure logarithms it will be found that the correct value of log 1351 is 130 6553, whilst the value found above is

180·656, an amount of error to which this method is liable, unless we carry the interpolations one place of figures further. It is on this account that, although the actual measurements of the records on the cylinder of the chronograph do not go beyond the $\frac{1}{1000}$th or $\frac{1}{2000}$th of a second, yet, in interpolation the consideration of $\frac{1}{10,000}$th or even a smaller part of a second is retained, in order to give greater facilities for the adjustment of column $\Delta_2$ by the application of small corrections, and consequently greater accuracy when the time is expressed in seconds carried to three places of decimals.

The above is an exact representation of the method adopted when the utmost care is required in calculating the time occupied by the projectile in passing from screen to screen. First of all, from some line drawn on the paper parallel to the axis of the cylinder, the clock records are measured and placed in one column; and the screen records are measured and placed in another column. The measurements for the clock should extend over five or six seconds, and overlap the screen records both ways. Both columns are then differenced as in (I). Small corrections are made, where needed, to introduce regularity in the column $\Delta_2$, and the figures are carried one place of decimals further than the measurements. The screen marks thus adjusted are left till a proper time-table has been constructed. By interpolation as in (III), the distance described by the clock tracing point at the end of every half-beat is found. By another interpolation, the space described by the clock marker at the end of every $\frac{1}{10}$th of a beat is found, as in (IV). In all cases it has appeared at this point, that the column of second differences was nothing, or extremely small, so that the rule of proportional parts could be used to determine fractions of a second less than $\frac{1}{10}$th of a second. This completes the time-table. We have now to determine by its help, the times corresponding to the spaces described by the screen tracing point when the first, second, &c. screen was passed: thus, referring to Round 1, we find that

the corrected reading for the sixth screen was...................... 41·325
and referring to the time-table we find corresponding to beat 5·4 41·105
·220

with a difference of 771 corresponding to $\frac{1}{10}$th of a beat. Therefore the sixth screen was passed by the shot at $5{\cdot}4000 + \frac{1}{10}\,\frac{220}{771} = 5{\cdot}4285$ in beats of the clock.

The necessity for the introduction of small corrections may spring from one or more of the following sources. The reading off from the original records cannot be exactly true. The thread may bend at one screen rather more than at another before breaking. The distance apart of the screens may not be exactly equal. The supporting poles may not be quite vertical. The projectile may strike a thread directly at one screen, at the next it may pass midway between the two threads. Now if some errors be constant throughout the experiment, and if others enter only occasionally into the records, they are of no moment, because the constant errors vanish, and the occassional errors are corrected in the process of differencing. The only errors to be feared are those which follow some law, for if there were such, they would combine with the law of motion of the projectile and so escape detection. But there is little danger of such errors in the use of the chronograph here described, for the records of the clock and of the screens are made under *precisely the same conditions.* With ten screens, there will be an absolute error, greater or less, in the records of the first and last screen, but whatever be the amount of error in the records of the intermediate screens, there can be no gain or loss of time. If one interval be a little too short, the next will be a little too long, and what is taken from one must be added to the other. The problem is to distribute the small errors between narrow limits. But suppose the nine intervals of time were measured by nine independent instruments, there would be an error, greater or less, at the beginning and end of every interval, making eighteen. And besides this there would be the nine different rates of the measurers of time, and various sources of error peculiar to the construction of so many independent instruments.

Eighteen rounds were fired to test the chronograph. No particular care was taken to examine the form and weight of the shot. The bore of the gun was about 3 inches, and the weight of the projectile about 12 lbs.

ROUND 1, November 23, 1865.

| Clock. | | | | | | Clock. | | | |
|---|---|---|---|---|---|---|---|---|---|
| Beat. | Reading. | Correction. | Corrected Reading. | $\Delta_1$ | $\Delta_2$ | Beat. | | $\Delta_1$ | $\Delta_2$ |
| 1 | 6·880 | + ·001 | 6·881 | + 7·828 | | 3·0 | 22·509 | + 3·889 | |
| 2 | 14·710 | − ·001 | 14·709 | + 7·800 | − 28 | ·5 | 26·398 | + 3·881 | − 8 |
| 3 | 22·510 | − ·001 | 22·509 | + 7·770 | − 30 | 4·0 | 30·279 | + 3·873 | − 8 |
| 4 | 30·290 | − ·011 | 30·279 | + 7·739 | − 31 | ·5 | 34·152 | + 3·866 | − 7 |
| 5 | 38·020 | − ·002 | 38·018 | + 7·709 | − 30 | 5·0 | 38·018 | + 3·858 | − 8 |
| 6 | 45·725 | + ·002 | 45·727 | + 7·682 | − 27 | ·5 | 41·876 | + 3·851 | − 7 |
| 7 | 53·410 | − ·001 | 53·409 | + 7·651 | − 31 | 6·0 | 45·727 | | |
| 8 | 61·060 | ·000 | 61·060 | | | | | | |

| Clock. | | | Screens. | | | | | |
|---|---|---|---|---|---|---|---|---|
| Beat. | Time Table. | $\Delta_1$ | No. of Screen. | Reading. | Correction. | Corrected Reading. | $\Delta_1$ | $\Delta_2$ |
| 4·5 | 34·152 | + ·774 | 1 | 36·190 | −·002 | 36·188 | + 1·007 | |
| ·6 | 34·926 | + ·773 | 2 | * * * | * * | 37·195 | + 1·017 | +10 |
| ·7 | 35·699 | + ·773 | 3 | 38·210 | +·002 | 38·212 | + 1·028 | +11 |
| ·8 | 36·472 | + ·773 | 4 | 39·240 | ·000 | 39·240 | + 1·038 | +10 |
| ·9 | 37.245 | + ·773 | 5 | 40·280 | −·002 | 40·278 | + 1·047 | + 9 |
| 5·0 | 38·018 | + ·772 | 6 | 41·330 | −·005 | 41·325 | + 1·054 | + 7 |
| ·1 | 38·790 | + ·772 | 7 | 42·330 | +·049 | 42·379 | + 1·060 | + 6 |
| ·2 | 39·562 | + ·772 | 8 | 43·440 | −·001 | 43·439 | + 1·066 | + 6 |
| ·3 | 40·334 | + ·771 | 9 | 44·500 | +·005 | 44·505 | + 1.071 | + 5 |
| ·4 | 41.105 | + ·771 | 10 | 45·580 | −·004 | 45·576 | | |
| 5·5 | 41·876 | + ·771 | | | | | | |
| ·6 | 42·647 | + ·770 | | | | | | |
| ·7 | 43·417 | + ·770 | | | | | | |
| ·8 | 44.187 | + ·770 | | | | | | |
| ·9 | 44·957 | + ·770 | | | | | | |
| 6·0 | 45·727 | | | | | | | |

By the help of the time-table the corrected screen readings are to be converted into time measured by beats of the clock. As the clock used did not beat seconds, a further calculation is required in this case to convert beats into seconds. As the period from which we measure time is quite arbitrary—if we subtract the time of passing the first screen from the times of passing the other screens, we obtain a table giving the time occupied by the shot in passing from the first to any succeeding screen. The whole must be differenced as a test of the accuracy of the work, and any necessary correction must be introduced. The first column of differences ($\Delta_1$) represents the time of passing from screen to screen. If the clock used, as is intended, beat seconds, and if it be found that during the course of any set of experiments it does not keep exact time, but gains or loses 1 in 1000, say, then the times last found will require to be diminished or increased 1 in 1000. A small correction of this kind was required, differing slightly in Rounds 1-9 and 10-18. If the clock had been perfectly adjusted, the length of a beat would have been 0·"81586.

| No. of Screen. | Corrected Screen Reading. | Beats of Clock. | Beats of Clock. | Seconds. | $\Delta_1$ | $\Delta_2$ |
|---|---|---|---|---|---|---|
| | | | | " | | |
| 1 | 36·188 | 4·7633 | 0·0000 | 0·0000 | + 1064 | |
| 2 | 37·195 | 4·8935 | 0·1302 | 0·1064 | + 1077 | +13 |
| 3 | 38·212 | 5·0252 | 0·2619 | 0·2141 | + 1088 | +11 |
| 4 | 39·240 | 5·1583 | 0·3950 | 0·3229 | + 1099 | +11 |
| 5 | 40·278 | 5·2927 | 0·5294 | 0·4328 | + 1109 | +10 |
| 6 | 41·325 | 5·4284 | 0·6651 | 0·5437 | + 1119 | +10 |
| 7 | 42·379 | 5·5652 | 0·8019 | 0·6556 | + 1126 | + 7 |
| 8 | 43·439 | 5·7029 | 0·9396 | 0·7682 | + 1132 | + 6 |
| 9 | 44·505 | 5.8413 | 1·0780 | 0·8814 | + 1136 | + 4 |
| 10 | 45·576 | 5·9804 | 1·2171 | 0·9950 | | |

For the remaining 10 rounds it will be sufficient to give the clock and screen readings and their corrections.

### ROUND 2.

| Clock. | | | Screens. | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Reading. | Correction. | Corrected Reading. | No. | Reading. | Correction. | Corrected Reading. | Time. | $\Delta_1$ | $\Delta_2$ |
| | | | | | | | " | | |
| 5·030 | ·000 | 5·030 | 1 | 32·993 | − ·001 | 32·992 | 0·0000 | +1045 | |
| 10·700 | + ·005 | 10·705 | 2 | * * * | * | 33·713 | ·1045 | +1053 | + 8 |
| 16·370 | + ·001 | 16·371 | 3 | 34·435 | + ·004 | 34·439 | ·2098 | +1062 | + 9 |
| 22·030 | − ·002 | 22·028 | 4 | 35·175 | − ·004 | 35·171 | ·3160 | +1072 | +10 |
| 27·680 | − ·003 | 27·677 | 5 | 35·905 | + ·005 | 35·910 | ·4232 | +1087 | +15 |
| 33·320 | ·000 | 33·320 | 6 | 36·665 | − ·005 | 36·660 | ·5319 | +1103 | +16 |
| 38·950 | + ·007 | 38·957 | 7 | 37·425 | − ·005 | 37·420 | ·6422 | +1116 | +13 |
| 44·590 | ·000 | 44·590 | 8 | 38·190 | ·000 | 38·190 | ·7538 | +1132 | +16 |
| | | | 9 | 38·970 | ·000 | 38·970 | ·8670 | +1148 | +16 |
| | | | 10 | 39·760 | ·000 | 39·760 | ·9818 | | |

## ROUND 5.

| Clock. | | | Screens. | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Reading. | Correction. | Corrected Reading. | No. | Reading. | Correction. | Corrected Reading. | Time. | Δ₁ | Δ₂ |
| | | | | | | | ″ | | |
| 20·860 | − ·004 | 20·856 | 1 | 26·190 | − ·003 | 26·187 | 0·0000 | | |
| 36·370 | + ·004 | 36·374 | 2 | * * * | * | 28·171 | ·1046 | +1046 | |
| 51·850 | ·000 | 51·850 | 3 | 30·170 | + ·006 | 30·176 | ·2102 | +1056 | +10 |
| 67·280 | ·000 | 67·280 | 4 | 32·210 | − ·008 | 32·202 | ·3170 | +1068 | +12 |
| | | | 5 | 34·250 | − ·002 | 34·248 | ·4248 | +1078 | +10 |
| | | | 6 | 36·290 | + ·023 | 36·313 | ·5338 | +1090 | +12 |
| | | | 7 | 38·400 | 000 | 38·400 | ·6438 | +1100 | +10 |
| | | | 8 | 40·510 | + ·002 | 40·512 | ·7553 | +1115 | +15 |
| | | | 9 | 42·650 | 000 | 42·650 | ·8682 | +1129 | +14 |
| | | | 10 | * * * | * | * * * | | | |

## ROUND 7, November 29, 1865.

| Clock. | | | Screens. | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Reading. | Correction. | Corrected Reading. | No. | Reading. | Correction. | Corrected Reading. | Time. | Δ₁ | Δ₂ |
| | | | | | | | ″ | | |
| 35·850 | + ·004 | 35·854 | 1 | 65·860 | − ·006 | 65·854 | 0·0000 | | |
| 53·560 | ·000 | 53·560 | 2 | 68·100 | − ·007 | 68·093 | ·1033 | +1033 | |
| 71·200 | + ·009 | 71·209 | 3 | 70·360 | + ·007 | 70·367 | ·2087 | +1054 | +21 |
| 88·830 | − ·015 | 88·815 | 4 | 72·670 | + ·004 | 72·674 | ·3157 | +1070 | +16 |
| 106·380 | + ·009 | 106·389 | 5 | * * * | * | 75·007 | ·4240 | +1083 | +13 |
| | | | 6 | 77·360 | ·000 | 77·360 | ·5333 | +1093 | +10 |
| | | | 7 | 79·720 | + ·009 | 79·729 | ·6433 | +1100 | + 7 |
| | | | 8 | 82·110 | + ·006 | 82·116 | ·7541 | +1108 | + 8 |
| | | | 9 | 84·530 | − ·007 | 84·523 | ·8659 | +1118 | +10 |
| | | | 10 | 86·950 | ·000 | 86·950 | ·9787 | +1128 | +10 |

## ROUND 10, December 12, 1865.

| Clock. | | | Screens. | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Reading. | Correction. | Corrected Reading. | No. | Reading. | Correction. | Corrected Reading. | Time. | Δ₁ | Δ₂ |
| | | | | | | | ″ | | |
| 18·140 | − ·002 | 18·138 | 1 | 38·960 | + ·002 | 38·962 | 0·0000 | | |
| 26·290 | + ·005 | 26·295 | 2 | 40·000 | − ·002 | 39·998 | ·1054 | +1054 | |
| 34·400 | − ·009 | 34·391 | 3 | 41·040 | + ·001 | 41·041 | ·2117 | +1063 | + 9 |
| 42·430 | − ·003 | 42·427 | 4 | * * * | * | 42·092 | ·3189 | +1072 | + 9 |
| 50·400 | + ·006 | 50·406 | 5 | * * * | * | 43·154 | ·4273 | +1084 | +12 |
| 58·320 | + ·011 | 58·331 | 6 | 44·240 | − ·014 | 44·226 | ·5368 | +1095 | +11 |
| | | | 7 | 45·310 | ·000 | 45·310 | ·6477 | +1109 | +14 |
| | | | 8 | 46·410 | − ·005 | 46·405 | ·7600 | +1123 | +14 |
| | | | 9 | * * * | * | 47·513 | ·8736 | +1136 | +13 |
| | | | 10 | 48·640 | − ·005 | 48·635 | ·9884 | +1148 | +12 |

## ROUND 11.

| Clock. | | | Screens. | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Reading. | Correction. | Corrected Reading. | No. | Reading. | Correction. | Corrected Reading. | Time. | Δ₁ | Δ₂ |
| | | | | | | | ″ | | |
| 9·010 | ·000 | 9·010 | 1 | 53·710 | + ·006 | 53·716 | 0·0000 | | |
| 21·440 | + ·002 | 21·442 | 2 | 55·290 | − ·006 | 55·284 | ·1047 | +1047 | |
| 33·820 | − ·004 | 33·816 | 3 | 56·870 | + ·004 | 56·874 | ·2110 | +1063 | +16 |
| 46·130 | ·000 | 46·130 | 4 | 58·490 | − ·004 | 58·486 | ·3188 | +1078 | +15 |
| 58·390 | − ·008 | 58·382 | 5 | 60·120 | ·000 | 60·120 | ·4281 | +1093 | +15 |
| 70·570 | + ·002 | 70·572 | 6 | 61·770 | + ·002 | 61·772 | ·5387 | +1106 | +13 |
| 82·710 | − ·010 | 82·700 | 7 | * * * | * | 63·440 | ·6505 | +1118 | +12 |
| | | | 8 | 65·120 | ·000 | 65·120 | ·7630 | +1125 | + 7 |

## ROUND 13.

| CLOCK. | | | SCREENS. | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Reading. | Correction. | Corrected Reading. | No. | Reading. | Correction. | Corrected Reading. | Time. ″ | Δ₁ | Δ₂ |
| 0·200 | − ·010 | 0·190 | 1 | 30·780 | + ·004 | 30·784 | 0·0000 | +1051 | |
| 9·880 | + ·001 | 9·881 | 2 | 32·020 | − ·002 | 32·018 | ·1051 | +1061 | +10 |
| 19·540 | − ·002 | 19·538 | 3 | 33·260 | + ·004 | 33·264 | ·2112 | +1071 | +10 |
| 29·170 | − ·008 | 29·162 | 4 | 34·520 | + ·001 | 34·521 | ·3183 | +1083 | +12 |
| 38·750] | + ·006 | 38·756 | 5 | 35·790 | + ·002 | 35·792 | ·4266 | +1096 | +13 |
| 48·320 | + ·003 | 48·323 | 6 | 37·080 | − ·002 | 37·078 | ·5362 | +1109 | +13 |
| 57·870 | − ·004 | 57·866 | 7 | 38·380 | − ·002 | 38.378 | ·6471 | +1121 | +12 |
| | | | 8 | 39·690 | + ·001 | 39·691 | ·7592 | +1131 | +10 |
| | | | 9 | 41·010 | + ·007 | 41·017 | ·8723 | +1143 | +12 |
| | | | 10 | 42·360 | − ·005 | 42·355 | ·9866 | | |

## ROUND 15.

| CLOCK. | | | SCREENS. | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Reading. | Correction. | Corrected Reading. | No. | Reading. | Correction. | Corrected Reading. | Time. ″ | Δ₁ | Δ₂ |
| 3·100 | − ·004 | 3·096 | 1 | 61·220 | − ·002 | 61·218 | 0·0000 | +1042 | |
| 15·330 | − ·002 | 15·328 | 2 | 62·760 | + ·001 | 62·761 | ·1042 | +1060 | +18 |
| 27·520 | + ·007 | 27·527 | 2 | 64·320 | + ·008 | 64·328 | ·2102 | +1078 | +18 |
| 39·690 | + ·003 | 39·693 | 4 | 65·920 | − ·004 | 65·916 | ·3175 | +1087 | +14 |
| 51·830 | − ·003 | 51·827 | 5 | 67·520 | + ·002 | 67·522 | ·4262 | +1097 | +10 |
| 63·930 | + ·002 | 63·932 | 6 | 69·150 | − ·006 | 69·144 | ·5359 | +1107 | +10 |
| 76·010 | ·000 | 76·010 | 7 | 70·770 | + ·009 | 70·779 | ·6466 | +1114 | + 7 |
| | | | 8 | 72·430 | − ·005 | 72·425 | ·7580 | | |

## ROUND 16.

| CLOCK. | | | SCREENS. | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Reading. | Correction. | Corrected Reading. | No. | Reading. | Correction. | Corrected Reading. | Time. ″ | Δ₁ | Δ₂ |
| 2·720 | ·000 | 2·720 | 1 | 52·460 | ·000 | 52·460 | 0·0000 | +1050 | |
| 18·010 | ·000 | 18·010 | 2 | 54·410 | − ·003 | 54·407 | ·1050 | +1061 | +11 |
| 33·260 | − ·005 | 33·255 | 3 | 55·970 | + ·406 | 56·376 | ·2111 | +1074 | +13 |
| 48·450 | + ·007 | 48·457 | 4 | 58·370 | − ·001 | 58·369 | ·3185 | +1088 | +14 |
| 63·620 | − ·002 | 63·618 | 5 | 60·380 | + ·006 | 60·386 | ·4273 | +1102 | +14 |
| 78·740 | ·000 | 78·740 | 6 | 62·420 | + ·007 | 62·427 | ·5375 | +1115 | +13 |
| | | | 7 | 64·500 | − ·008 | 64·492 | ·6490 | +1127 | +12 |
| | | | 8 | * * * | * | 66·580 | ·7617 | +1140 | +13 |
| | | | 9 | 68.690 | ·000 | 68·690 | ·8757 | +1151 | +11 |
| | | | 10 | 70·820 | ·000 | 70·820 | ·9908 | | |

## ROUND 17.

| CLOCK. | | | SCREENS. | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Reading. | Correction. | Corrected Reading. | No. | Reading. | Correction. | Corrected Reading. | Time. ″ | Δ₁ | Δ₂ |
| 10·830 | + ·008 | 10·838 | 1 | 55·370 | − ·002 | 55·368 | 0·0000 | +1049 | |
| 23·100 | − ·008 | 23·092 | 2 | 56·930 | + ·002 | 56·932 | ·1049 | +1065 | +16 |
| 35·320 | + ·008 | 35·328 | 3 | 58·520 | ·000 | 58·520 | ·2114 | +1079 | +14 |
| 47·540 | + ·004 | 47·544 | 4 | 60·140 | − ·011 | 60·129 | ·3193 | +1092 | +13 |
| 59·750 | − ·009 | 59·741 | 5 | 61·760 | − ·004 | 61·756 | ·4285 | +1104 | +12 |
| 71·910 | + ·008 | 71·918 | 6 | 63·440 | − ·038 | 63·402 | ·5389 | +1116 | +12 |
| 84·070 | + ·003 | 84·073 | 7 | 65·060 | + ·006 | 65·066 | ·6505 | +1131 | +15 |
| | | | 8 | * * * | * | 66·751 | ·7636 | +1147 | +16 |
| | | | 9 | 68·460 | − ·001 | 68·459 | ·8783 | +1164 | +17 |
| | | | 10 | 70·190 | + ·003 | 70·193 | ·9947 | | |

### ROUND 18.

| CLOCK. | | | SCREENS. | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Reading. | Correction. | Corrected Reading. | No. | Reading. | Correction. | Corrected Reading | Time. | Δ1 | Δ2 |
| | | | | | | | ″ | | |
| 46·350 | ·000 | 46·350 | 1 | 57·270 | + ·001 | 57·271 | 0·0000 | +1057 | |
| 54·790 | + ·010 | 54·800 | 2 | 58·360 | + ·003 | 58·363 | ·1057 | +1066 | + 9 |
| 63·260 | − ·010 | 63·250 | 3 | 59·470 | − ·004 | 59·466 | ·2123 | +1077 | +11 |
| 71·700 | ·000 | 71·700 | 4 | 60·580 | − ·001 | 60·579 | ·3200 | +1087 | +10 |
| | | | 5 | 61·700 | + ·003 | 61·703 | ·4287 | +1098 | +11 |
| | | | 6 | 62·840 | − ·002 | 62·838 | ·5385 | +1110 | +12 |
| | | | 7 | 63·980 | + ·005 | 63·985 | ·6495 | +1123 | +13 |
| | | | 8 | * * * | * | 65·146 | ·7618 | +1136 | +13 |
| | | | 9 | 66·320 | + ·001 | 66·321 | ·8754 | +1151 | +15 |
| | | | 10 | 67·510 | − ·001 | 67·509 | ·9905 | | |

As these experiments were made merely to test the instrument, the forms and weights of the shot were not examined, consequently nothing would be gained by attempting to find the law of resistance in each of the preceding cases. This method of proceeding, or something equivalent, will be necessary when that law is sought for, because the shot do not pass the first screen with the *same* velocity. At present it will be sufficient to find the mean time occupied by the ball in passing from the first to each of the following screens. We are thus able to answer, *from experiment alone,* the practical question, what is the *average* velocity of a 12 lb. shot fired from the given gun with the given charge, at any point between 120 feet and 1000 feet from the gun?

| Round | Screen 1 | Screen 2 | Screen 3 | Screen 4 | Screen 5 | Screen 6 | Screen 7 | Screen 8 | Screen 9 | Screen 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ |
| 1 | 0·0000 | 0·1064 | 0·2141 | 0·3229 | 0·4328 | 0·5437 | 0·6556 | 0·7682 | 0·8814 | 0·9950 |
| 2 | ·0000 | ·1045 | ·2098 | ·3160 | ·4232 | ·5319 | ·6422 | ·7538 | ·8670 | ·9818 |
| 5 | ·0000 | ·1046 | ·2102 | ·3170 | ·4248 | ·5338 | ·6438 | ·7553 | ·8682 | * |
| 7 | ·0000 | ·1033 | ·2087 | ·3157 | ·4240 | ·5333 | ·6433 | ·7541 | ·8659 | ·9787 |
| 10 | ·0000 | ·1054 | ·2117 | ·3189 | ·4273 | ·5368 | ·6477 | ·7600 | ·8736 | ·9884 |
| 11 | ·0000 | ·1047 | ·2110 | ·3188 | ·4281 | ·5387 | ·6505 | ·7630 | * | * |
| 13 | ·0000 | ·1051 | .2112 | ·3183 | ·4266 | ·5362 | ·6471 | ·7592 | ·8723 | ·9866 |
| 15 | ·0000 | ·1042 | .2102 | ·3175 | ·4262 | ·5359 | ·6466 | ·7580 | * | * |
| 16 | ·0000 | ·1050 | ·2111 | ·3185 | ·4273 | ·5375 | ·6490 | ·7617 | ·8757 | ·9908 |
| 17 | ·0000 | ·1049 | ·2114 | ·3193 | ·4285 | ·5389 | ·6505 | ·7636 | ·8783 | ·9947 |
| 18 | ·0000 | ·1057 | ·2123 | ·3200 | ·4287 | ·5385 | ·6495 | ·7618 | ·8754 | ·9905 |
| Totals ... | ·0000 | 1·1538 | 2·3217 | 3·5029 | 4·6975 | 5·9052 | 7·1258 | 8·3587 | 7·8578 | 7·9065 |
| Divisors | ... | 11 | 11 | 11 | 11 | 11 | 11 | 11 | 9 | 8 |
| Means... | ·00000 | ·10489 | ·21106 | ·31845 | ·42705 | ·53684 | ·64780 | ·75988 | ·87309 | ·98831 |

| Distance from gun. ft. | Screen. | Average Time. | Δ1 | Δ2 |
|---|---|---|---|---|
| 120 | 1 | ″ 0·00000 | + 10489 | |
| 240 | 2 | ·10489 | + 10617 | + 128 |
| 360 | 3 | ·21106 | + 10739 | + 122 |
| 480 | 4 | ·31845 | + 10860 | + 121 |
| 600 | 5 | ·42705 | + 10979 | + 119 |
| 720 | 6 | ·53684 | + 11096 | + 117 |
| 840 | 7 | ·64780 | + 11208 | + 112 |
| 960 | 8 | ·75988 | + 11321 | + 113 |
| 1080 | 9 | ·87309 | + 11522 | + 201 |
| 1200 | 10 | ·98831 | | |

The irregularity at the 9th and 10th screens is caused by the failure of a few screens in rounds 5, 11, and 15. We can now, by means of a common interpolation formula calculate the average time occupied by the shot in moving over 20 feet, or $\frac{1}{6}$ of 120 feet, from each screen. And dividing 20 by this fraction of a second, we obtain the velocity of the shot at 130, 250, 370, &c. feet from the gun.

| Distance from gun. | Velocity. f.s. | $\Delta_1$ | $\Delta_2$ |
|---|---|---|---|
| ft. | | | |
| 130 | 1149·4 | −13·7 | |
| 250 | 1135·7 | −13·1 | +·6 |
| 370 | 1122·6 | −12·7 | +·4 |
| 490 | 1109·9 | −12·2 | +·5 |
| 610 | 1097·7 | −11·6 | +·6 |
| 730 | 1086·1 | −11·1 | +·5 |
| 850 | 1075·0 | −10·7 | +·4 |
| 970 | 1064·3 | | |

By interpolation we are now able to obtain from experiment *alone* the average velocity of the eleven shot at every ten feet from the gun, from 130 to 1000 feet. The result is

| Distance from gun. | Velocity. f.s. | Distance from gun. | Velocity. f.s. | Distance from gun. | Velocity. f.s. | Distance from gun. | Velocity. f.s. |
|---|---|---|---|---|---|---|---|
| ft. | | ft. | | ft. | | ft. | |
| 130 | 1149·4 | 360 | 1123·6 | 590 | 1099·7 | 820 | 1077·7 |
| 140 | 1148·2 | 370 | 1122·6 | 600 | 1098·7 | 830 | 1076·8 |
| 150 | 1147·1 | 380 | 1121·5 | 610 | 1097·7 | 840 | 1075·9 |
| 160 | 1145·9 | 390 | 1120·4 | 620 | 1096·8 | 850 | 1075·0 |
| 170 | 1144·8 | 400 | 1119·4 | 630 | 1095·8 | 860 | 1074·1 |
| 180 | 1143·6 | 410 | 1118·3 | 640 | 1094·8 | 870 | 1073·2 |
| 190 | 1142·5 | 420 | 1117·2 | 650 | 1093·8 | 880 | 1072·3 |
| 200 | 1141·3 | 430 | 1116·2 | 660 | 1092·9 | 890 | 1071·4 |
| 210 | 1140·2 | 440 | 1115·1 | 670 | 1091·9 | 900 | 1070·5 |
| 220 | 1139·0 | 450 | 1114·1 | 680 | 1090·9 | 910 | 1069·6 |
| 230 | 1137·9 | 460 | 1113·0 | 690 | 1089·9 | 920 | 1068·7 |
| 240 | 1136·8 | 470 | 1112·0 | 700 | 1089·0 | 930 | 1067·8 |
| 250 | 1135·7 | 480 | 1110·9 | 710 | 1088·0 | 840 | 1066·9 |
| 260 | 1134·6 | 490 | 1109·9 | 720 | 1087·1 | 950 | 1066·1 |
| 270 | 1133·5 | 500 | 1108·8 | 730 | 1086·1 | 960 | 1065·2 |
| 280 | 1132·4 | 510 | 1107·8 | 740 | 1085·2 | 970 | 1064·3 |
| 290 | 1131·3 | 520 | 1106·8 | 750 | 1084·3 | 980 | 1063·4 |
| 300 | 1130·2 | 530 | 1105·8 | 760 | 1083·3 | 990 | 1062·6 |
| 310 | 1129·1 | 540 | 1104·7 | 770 | 1082·4 | 1000 | 1061·7 |
| 320 | 1128·0 | 550 | 1103·7 | 780 | 1081·5 | | |
| 330 | 1126·9 | 560 | 1102·7 | 790 | 1080·5 | | |
| 340 | 1125·8 | 570 | 1101·7 | 800 | 1079·6 | | |
| 350 | 1124·7 | 580 | 1100·7 | 810 | 1078·7 | | |

Supposing that the average times occupied by the shot in travelling from the first to each succeeding screen were expressed in seconds carried to only three places of decimals, the second differences of the time would be nearly constant. If they were exactly constant, then the time $t$ occupied by the shot in passing from the first screen to a point $s$ feet from it, would be connected with $s$ by an equation of the form $t=as+bs^2$. This would give the velocity $v$ at that point $=(a+2bs)^{-1}$, and a retarding force varying as $v^3$.

If the average values of $s$ and $t$, determined by experiment at the 4th and 8th screens, be used to find $a$ and $b$, we have

$$\cdot 31845 = a \times 360 + b \times (360)^2,$$

$$\cdot 75988 = a \times 840 + b \times (840)^2,$$

which two equations give $a = \cdot 000,\ 869,\ 558,$

$$b = \cdot 000,\ 000,\ 0417,$$

$$\text{and } \therefore\ t = \cdot 000,\ 869,\ 558\,s + \cdot 000,\ 000,\ 0417\,s^2,$$

$$v = \frac{1000}{\cdot 869,\ 558, + \cdot 000,\ 0834s}.$$

Or if we measure the distance $s'$ from the gun we have $s'=s+120$ or $s=s'-120$,

$$v = \frac{1000}{\cdot 869,\ 588 + \cdot 000{,}0834\,(s'-120)}$$

$$= \frac{1000}{\cdot 859,\ 550 + \cdot 000,\ 0834s'},$$

and giving $s'$ the particular values 0, 100, 200, &c., we find the velocities of the shot at 0, 100, 200, &c. feet from the gun by the formula, which we may compare with those derived purely from experiment by interpolation, as given above

| Distance from gun. | Velocity by Formula. | Difference. | Velocity by Experiment. |
|---|---|---|---|
| ft. | f.s. | f.s. | f.s. |
| 0 | 1163·4 | * | * * * |
| 100 | 1152·2 | * | * * * |
| 200 | 1141·3 | 0·0 | 1141·3 |
| 300 | 1130·5 | − 0·3 | 1130·2 |
| 400 | 1119·9 | − 0·5 | 1119·4 |
| 500 | 1109·6 | − 0·8 | 1108·8 |
| 600 | 1099·4 | − 0·7 | 1098·7 |
| 700 | 1089·4 | − 0·4 | 1089·0 |
| 800 | 1079·6 | 0·0 | 1079·6 |
| 900 | 1070·0 | + 0·5 | 1070·5 |
| 1000 | 1060·5 | + 1·2 | 1061·7 |

It is evident that this table requires to be extended both ways, so as to give the law for higher and lower velocities. Thus initial velocities of 1200 f.s. or more, and lower velocities of 1100, 1050, 1000 f.s., &c., are required. In all these cases it must be remembered that our velocity is the horizontal velocity $\frac{dx}{dt}$ or $v_1$ of M. Didion.

To obtain a satisfactory table shewing the velocity of a given ball projected with the highest possible velocity, at all distances from the gun, I am satisfied that four good shots through ten screens with each particular charge would be amply sufficient. The shot should have exactly the same form of head, and be of precisely the same weight. One shot should be fired with a velocity 1200 f.s., then one with 1150 f.s., then one 1100 f.s., so far as the height of the screens would allow. The same series must be gone through three times more. In the subsequent treatment of a complete and standard experiment of this kind, it is evident that instead of measuring the time from the instant of passage of the first screen, it will be necessary to measure the time in each round of four, from the instant when each shot had a certain velocity.

A few hollow shot of the same external form might be fired with advantage. For low velocities it would be advisable to remove three or four of the middle screens.

A good test of the truth of the law deduced from experiments with various initial velocities would be obtained by firing a few rounds with the highest initial velocity through three screens near the gun, and three at a distance of 4500 or 6000 feet. The chronograph would give initial and final velocities and the time of flight. It would be advisable to try the experiments on the long range when the wind was blowing directly with and directly against the shot, also when the wind was blowing at right angles to the direction of the line of fire. Also the effect of the height of the barometer, and of the degree of moisture, on the resistance of the air might best be determined by a few shots on the long range when the air was perfectly still.

In the experiments just suggested, the law of initial velocity, so far as it depends upon the charge, would be determined with great precision. The results might also be treated in two ways, that is, so as to obtain both an average velocity at any distance from the gun, and the true law of the resistance of the air.

If *similar* forms of shot were used in experiments of the above kind, with bores of 3, 5, 7, 9, 11, and 13 inches, we should be able to extend, by interpolation, the laws obtained from these experiments for the above particular cases to all possible calibres from 3 to 13 inches. It would be an easy matter to bring all needful experiments of the above nature to a satisfactory conclusion within six months. When this work was once well done there could be no pretence for ever re-opening the question. Take, for instance, the table of velocities which has been given for one particular calibre and form of head. What more information can be required for the same gun and projectile after that table has been extended both ways with the same exactness, to the highest and lowest practical limits? The force of the charge of powder, taking account of quality and quantity, must be known in any practical case. Suppose that the initial velocity for a smaller charge, or a different powder, was 1120 f.s., and that it was required to find the velocity at a point 500 feet from the gun. Referring to the table, we find the velocity at 394 feet from the gun was 1120 f.s., and 500 feet farther on, or at 394 + 500 = 894 feet in the table, the velocity was 1071 f.s. So that a shot starting with an initial velocity of 1120 f.s. would have a velocity 1071 f.s. after it had travelled 500 feet through the air. All this information is derived purely from experiment, and is therefore free from any question as to the value of a formula or a theory. For a hollow shot, of the same external form as the solid shot used in the experiment, the air would exert precisely the same resistance for a given velocity; but its effect would be more sensible from the diminished mass of the body it acted upon. For a slight increase of length in the cylindrical part of a shot there is but a very slight increase of the resistance of the air.

Having determined the law of variation of the velocity of a cannon ball moving through the air, there remains the scarcely less important problem to find according to what law the velocity is communicated to the shot inside the bore of the gun by the action of the gas of the exploding powder. The solution of this problem will shew how the work is distributed inside the gun, and consequently the amount of stress upon the gun corresponding to any position of the shot, for a given quality and weight of powder. If one powder exploded more suddenly and completely than another, and if the weight of the charges were so adjusted as to give the same velocity when the shot leaves the gun; although the total work done would be the same, the law of distribution of the work would be very different in the two cases. The rapidly exploding powder would give a high initial pressure upon the ball, which would decrease rapidly as the ball moved, and become less towards the muzzle than that arising from the explosion of a slowly burning powder. The former would therefore be a more severe trial of the gun than the latter, although the same velocity of the ball would be obtained in the two cases. If powder could be manufactured and fired so as to keep up a *constant* pressure upon the ball from one end of the bore to the other, in such a case a given velocity would be obtained with the least possible strain upon the gun. The introduction of rifled ordnance has rendered the degree of rapidity of the explosion of powder a question of greater importance than it was when smooth bores only were used; for the effect of rifling must be to hinder the forward motion of the ball at the first, and consequently to increase the initial strain on the gun. The increasing spiral groove is therefore a great improvement on the uniform spiral; its form ought to be such as to give equal additions to the angular velocity of the shot in equal *times*, not in equal *spaces*. Consequently, as the velocity of translation of a shot depends upon the nature of the powder, the weight of the charge, and the weight of the ball, a change in any one of these would, in strictness, call for a different kind of rifling. But probably great nicety is not required, provided the twist is small at the bottom of the bore, and that it increases gradually up to such an angle at the muzzle as is required to give steadiness to the shot. The angular velocity necessary to give the requisite steadiness to shot of various lengths and calibres requires careful experimental treatment, because we do not know theoretically the exact forces which act upon the projectile.

The common formula for initial velocity is built upon the assumptions—

(1) That all the powder is converted into gas before the ball moves sensibly.

(2) That the pressure of the gas afterwards always varies as its density.

If these assumptions were true, it would not matter what air space was left between the powder and ball, or what was the number of shot put into the gun. For a given space occupied by the gas there could only be one pressure, and the only effect of either of the above experiments would be to increase the time during which that pressure acted. Experiments have shewn that the initial velocity of a shot will depend upon the point at which the charge is ignited, all other things remaining the same. And Daniel Bernouilli, who gave the common formula for the calculation of initial velocity in 1738, at that time expressed his belief that the whole charge was not fired instantaneously, and more than a century ago a Committee of the Royal Society, appointed to examine this question, came to the conclusion that the whole of the powder is *not* fired before the ball is sensibly moved from its place. Why then are we to retain a mathematical formula which is manifestly based on an erroneous assumption? The thorough contradiction between the results of this formula and of the experiments with the 70-pr. Armstrong and Whitworth guns with charges of 10 lbs., ought to be the death blow to the formula. The length of bore of the Armstrong muzzle-loader was 10 feet 3·25 inches, and length of cartridge, 12·81 inches. The ratio of weight of powder to weight of ball was ·1403; and the initial velocity was 1336 f.s. Substituting these values in the formula

$$V = \sqrt{\gamma \frac{\mu}{m + \frac{1}{3}\mu} \log \frac{M}{\mu}},$$

$$1836 = \sqrt{\gamma} \sqrt{\frac{\cdot 1403}{1 + \frac{1}{3} \times \cdot 1403} \log \frac{123\cdot 25}{12\cdot 81}}, \text{ which gives } \sqrt{\gamma} = 3680\cdot 3.$$

As the powder was the same in quality in the two cases $\sqrt{\gamma}$ is a constant. Now apply the formula to calculate the initial velocity for the Whitworth gun, the bore of which was 9 feet 11 inches = 119 inches long. The length of cartridge was 14·5 inches, and the ratio of weight of powder to weight of shot was ·1427. We find by the formula the initial velocity

$$= 3680\cdot 3 \times \sqrt{\frac{\cdot 1427}{1 + \frac{1}{3} \times \cdot 1427} \log \frac{119}{14\cdot 5}} = 1298\cdot 7 \text{ f.s.},$$

whilst the velocity found by experiment was 1338 f.s. Thus the formula gives for the Whitworth gun a velocity *decidedly less* than that for the Armstrong, whereas experiment gives a velocity *slightly greater*. The formula gives 1298·7 feet as the velocity at the gun, whereas experiment shews that this would be the velocity at a point about 600 feet from the gun.

Under these circumstances it is quite plain that this formula is perfectly useless in determining the stress upon the gun. Captain Rodman used his pressure gauge in very numerous experiments, but as might have been expected from the shortness of the time during which the pressures were applied, the instrument did not give results sufficiently regular. Major Navez has proposed to use his pendulum to measure the times occupied by the shot in traversing different parts of the bore of the gun, but inasmuch as these intervals of time must be very short, and as every measurement must depend upon the action of three electro-magnets, it may very naturally be doubted whether the results would be of any value. The only method which appears to me to hold out any prospect of success, is one to which Captain Rodman gave a very partial trial. He mounted a gun in the old gun pendulum. A cylinder was placed with its axis parallel to the bore of the gun. When the gun recoiled it drew back a tracing point which marked a straight line on the surface of the cylinder at rest. If, whilst the gun was stationary, the cylinder was made to rotate, the tracing point described a circle on the surface of the cylinder. If now the cylinder be made to rotate rapidly, and left to itself, and if the gun be fired, the point will trace out a curve on the surface of the cylinder, the ordinate of which will represent the time occupied by the gun in recoiling through a space equal to the abscissa. The *law* of the work done to produce the recoil thus becomes known. By measuring the initial velocity of the shot, the total amount of the work done by the powder is found. The law of the pressure of the gas would be found with greater exactness if the tracing point was connected with the projectile. This method of experimenting seems to be the only one likely to give a satisfactory result, for if the rotation of the cylinder be uniform for the very short time the explosion is in progress, there can be no doubt that the co-ordinates of the diagram would give us a connexion between the time and space, and knowing the weight and velocity of the moving body at any one point, we can tell the amount of pressure that was acting upon it at any instant, or for any position. From what has been said, it is evidently of great importance in the proof of gunpowder, that besides the trial for initial velocity, there should be some test of the stress thrown on the gun.

The law of penetration of iron plates by hardened steel shot is another purely mathematical question. When a particular form of head of shot has been decided upon, and when a satisfactory method of hardening the shot has been discovered, it will be an easy matter to determine the laws which govern the perforation of iron plates. The quantities to be connected are the velocity, the weight, and the diameter of the shot, and the thickness of the iron plates. It is probable that a series of experiments conducted with plates of tolerably good iron of a uniform quality would be sufficient to afford all necessary information. A few comparative experiments might be made with plates of different qualities of iron but of equal thicknesses.

When opportunity offers, it is intended to repeat and extend the experiments of Robins and Hutton with the whirling machine. It is evident that this chronograph would give the exact time of each revolution of the machine from rest up to the attainment of uniform motion. Each experiment would thus afford a determination of the *law* of resistance of the air to the motion of the particular form of body. If a sufficient fall could be obtained it would be easy to make some very exact experiments on the force of gravity and the resistance of the air at low velocities. Comparative experiments on the velocity of shot and the velocity of the sound of the explosion of the gun may be made, provided the concussion of the air can be caused to break for a moment a galvanic current. If this can be done, the time occupied by the sound in passing over every successive 100 feet can be measured; and when the laws of resistance of the air have been determined, it is probable that the simplest instrument that can be used to find mere initial velocities will be one which measures the comparative velocities of the shot and the sound of the explosion.

The chronograph here described was contrived solely with a view to the satisfactory measurement of the velocities of bodies, and the determination of *the resistance of* the air to their motion, and without the slightest concern about the cost or weight of the instrument. Possibly an instrument-maker would construct a chronograph for £100, and a clock for £25. The shed to hold the two might cost £50. Thus the cost would be about £175; but to cover all contingencies I would double the above sum and estimate the cost at £350. But, on the other hand, the saving in powder and shot must be considered in making a connected series of experiments. For instance, the chronograph makes 10 records, or more, for a single shot. And further we must allow the value of each record to be at least 10 times the value of that made by an instrument which measures only one interval. So that it seems to me that the new chronograph, in economy of ammunition, compared with any chronograph in use, may be placed at 100 to 1. Extravagant as this estimate may appear, it is really understated. I maintain that any one of the eleven shots already described gives more exact information than all previous experiments made with the same gun fired with the same charge. But there need be no difficulty about cost, for the instrument is already constructed complete in all its parts, and ready to make all experiments required for a perfect system of gunnery in the course of a few weeks. To shew the capability of the instrument, we will suppose the chronograph placed in some central position at Shoeburyness, and provided with wires running to six different ranges, and that no less than six different parties wished to conduct experiments at the same time. Five or ten minutes at least are required to repair a set of screens, but only a few seconds are required to complete the experiment when the gun is ready to be fired. Each party then, as it was prepared, would have to telegraph that it was ready. Communications would be made so as to send the galvanic current through the proper screens, the fly-wheel would be put in motion, the signal to fire would be given, and the experiment would be completed. By each party keeping an account of the exact time of the firing of their guns, they would be able to claim their own records at the end of the day without the slightest difficulty or confusion.

I trust that the reader who has had the patience to follow me through this long account of the chronograph, and of the way of using it, will be aware that many questions have been put to me in the course of the last two years, to which I was not able to reply in a brief and satisfactory manner without opening the whole subject. I hope also that they who have urged objections to my methods will be satisfied that I have provided against all real difficulties, and that where I could not overcome a difficulty I have found a way of avoiding it. The screens are perfectly satisfactory, and looking at the instrument as it stands, I cannot see any point in which greater simplicity is practicable or desirable. Certainly if the fly-wheel could be made to revolve with *perfect uniformity* and with a *known velocity*, the labour of forming a time-table for each experiment would be avoided, but no sensible increase of accuracy would be obtained in the result. Undoubtedly there is trouble attending the reduction of these as well as of all other observations where the greatest attainable precision is required. All this trouble I am ready to take upon myself, if in other respects every facility is given me for completing the proposed work.

In conclusion, I must remind the reader that the questions raised in this tract are purely mathematical and mechanical. I have found it necessary to endeavour to clear the way by shewing the exact value of some formulæ and instruments which have gradually come into use from the absence of something better and more trustworthy. It is proposed to replace the former doubtful or erroneous conclusions by the results of exact experiments made with the guns and projectiles actually in the service, and so to connect them by comparison and interpolation, that all doubt or uncertainty of the smallest degree shall be utterly excluded. When these facts have been arrived at, all that is necessary for practical purposes will have been obtained, and a theory must be based on these facts. Afterwards the application of the results to supply the wants of the army and navy must pass into the hands of military and naval officers, assisted by chemists and mechanical engineers. And I shall feel that my time has been well employed if my results can be turned to account in providing for the defence of our rights and our home. Although I am convinced that a sincere desire for peace pervades all classes of this nation, yet that is no security for the continued enjoyment of the blessings of peace. It is possible that, like Denmark, we may have war forced upon us. The most likely way to keep the peace is to let it be clearly understood, by all whom it may concern, that we know perfectly well how to use the immense power which we, as a nation, possess. And if as a last resource, and the one which, on the whole, offers the least of evils, we are obliged to send forth brave men to fight in a just cause, we shall have the satisfaction of knowing that we have supplied them with every help which the nation can provide for them in the discharge of their trying duty.

July 3, 1866.

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